3D LATTICE PANELS BASED ON THE CONCAVE POLYHEDRA OF THE SECOND SORT: IDEAS FOR ARCHITECTURAL ORNAMENTS

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Abstract: Using the concave polyhedra of the second sort, we are creating spatial structures in the shape of lattice panels to be applied in architecture. The procedure is based on a rectangular (or, less often, a polar) array of identical representatives of the concave polyhedra that include: concave antiprisms, concave cupolae and concave pyramids of the second sort. The selected representative, as a unit cell, can be arrayed so to touch the adjacent cells by vertex, by edge or by face. Thereby, they are forming 3D lattice, similar to the 2D lattices patterns. We are using a single layer of these structures to form a shape most convenient for architectural usage, which is a shape of a panel. These 3D lattice panels are proposed to be used as brise-soleil, room dividers, fences, etc. The additional layer of visual design when using such a panels is accomplished with the shadows they cast, depending on the time and day of the year. 3D shape emphasizes the play of light and shadow, so these lattice panels can have a significant role as an element of decoration, i.e. architectural ornament.
Keywords: deltahedron, lattice, antiprism, triangle, tessellation, architecture, brise soleil

INTRODUCTION

Concave polyhedra of the second sort (abbreviated: C-II-n) constitute of a group of polyhedra formed over a regular n-sided base polygon, having a deltahedral lateral surfaces. This group includes: Concave cupolae of the second sort:CC-II-nM and CC-II-nm[1], Concave pyramids of the second sort**:** CP-II-nM, CP II-nm[2], and CP-II-nB[3] and Concave antiprisms of the second sort: CA-II-nM and CA-II-nm[4].

The common feature of these solids is that their lateral surfaces consist of a double row of equilateral triangles which can be assembled in two ways, making two different solids' heights: major (C-II-nM) and minor (C-II-nm). The geometrical regularities and a high level of symmetry that characterizes these polyhedra, makes them suitable for joining and combining, so they can be arrayed infinitely in space, in x, y and z direction forming 3D lattice structures.

In the previous research, we have dealt with combining and compatibility of these solids, and even their modularity[5] in case their bases are congruent. For certain representatives of these solids, the complete overlap of some of their lateral faces occurs, so 3D tessellations can be formed.

In this research, we are using the experience gained in the previous ones, together with the results, to propose a new set of 3D shapes organized as lattice panels. The shape of panel is chosen because it is one of the most common and also most universal elements, ubiquitous in architectural design. The lattice structure is chosen because it brings the geometric ornament to its expression, having the regularities and simplicity in one hand, and the complexity and eye-appeal in the other.

The overview of the representatives of the concave polyhedra of the second sort that can be used as unit cell in forming the 3D lattice panels is given below.

1. Concave cupolae of the second sort

Concave cupolae of the second sort (CC II-nM and CC II-nm)[6] comprise of 14 representatives in total. Each representative of these polyhedra consists of two regular polygons in the parallel planes: n sided and 2n-sided, where 3≤n≤10, connected by deltahedral lateral surface.

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1 M. Obradović and S. Mišić,"Concave Regular Faced Cupolae of Second Sort", in: *Proceedings of 13th ICGG*, ed. G. Weiss, Dresden, 2008, El. Book: 1–10

2 M. Obradović, S. Mišić and B. Popkonstantinović,"Concave Pyramids of Second Sort –The Occurrence, Types, Variations", in: *Proceedings of the 4th International Scientific Conference on Geometry and Graphics*, *moNGeometrija 2014*, ed. S. Krasić, Vol 2. M. Obradović, S. Mišić, B. Popkonstantinović, M. Petrović, B. Malešević, R. Obradović: Vlasina, 2014, 157–168.

3 M. Obradović, S. Mišić and B. Popkonstantinović,"Variations of Concave Pyramids of Second Sort with an Even Number of Base Sides", *Journal of Industrial Design and Engineering Graphics (JIDEG) – The SORGING Journal, Volume 10, Special Issue, Fascicle 1*, ed. D. Marin, Brasov, 2015, 45–50.

4 M. Obradović**,** B. Popkonstantinović and S. Mišić, "On the Properties of the Concave Antiprisms of Second Sort", *FME Transactions*. Vol. 41 No 3, (Belgrade), 2013, 256–263.

5 M. Obradović, "Modularity of concave polyhedra of the second sort with octagonal bases", in: *Proceedings of the 18th International Conference on Geometry and Graphics. ICGG 2018. Advances in Intelligent Systems and Computing*, Vol. 809. No1. Ed. L. Cocchiarella, Milan, 2018, 942–954.

6 M. Obradović and S.Mišić, "Concave Regular Faced Cupolae of Second Sort", in: *Proceedings of 13th ICGG*, ed. G. Weiss, Dresden, 2008, El. Book: 1–10

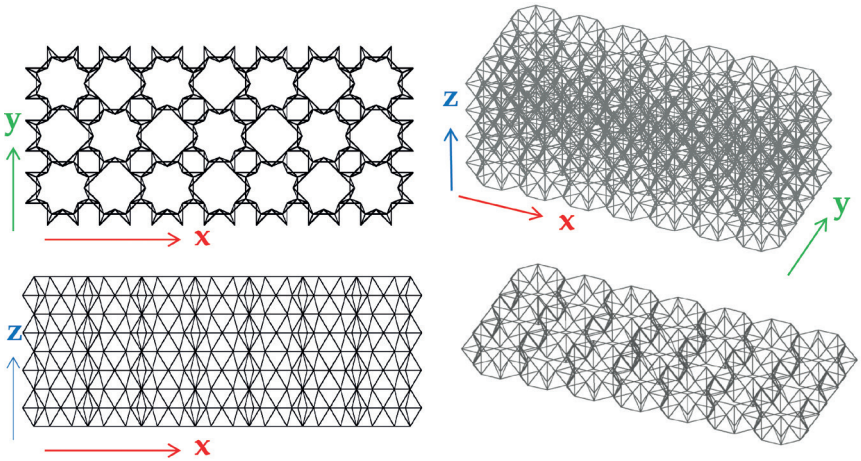

Fig. 1

2. Concave pyramids of the second sort

Concave pyramids of the second sort, abbreviated: CP-II-nM, CP-II-nm[7] and CP-II-nB[8], comprise of 17 representatives. Their double-rowed deltahedral lateral surface is assembled over a regular polygon, which can be from a triangle to a nonagon. They can be formed with a larger number of triangles (5n) in the lateral surface, which applies to each basis of $3 \leq n \leq 9$ sides.

They can be also formed with a lesser number of triangles (3n) in the lateral surface, which applies only to the base polygons with an even number of sides, $n \in \{6, 8, 10\}$.

3. Concave antiprisms of the second sort

Concave antiprisms of the second sort (CA-II-nM and CA-II-nm)[9] can be formed over any regular polygon, with the infinite number of members. They are formed with two congruent regular polygons in the parallel planes, connected by the lateral deltahedral surface.

3D LATTICE STRUCTURE AS A SPATIAL GRID

3D lattices are three-dimensional structures made up of nodes (polyhedron vertices) and struts (polyhedron edges) that form a spatial grid[10], similarly to the 2D lattices. Such a lattice replaces solid fill, making

7 M. Obradović, S. Mišić and B. Popkonstantinović,"Concave Pyramids of Second Sort –The Occurrence, Types, Variations", in: *Proceedings of the 4th International Scientific Conference on Geometry and Graphics*, *moNGeometrija 2014*, ed. S. Krasić, Vol 2. M. Obradović, S. Mišić, B. Popkonstantinović, M. Petrović, B. Malešević, R. Obradović: Vlasina, 2014, 157–168.

8 M. Obradović, S. Mišić and B. Popkonstantinović,"Variations of Concave Pyramids of Second Sort with an Even Number of Base Sides", *Journal of Industrial Design and Engineering Graphics (JIDEG) – The SORGING Journal, Volume 10, Special Issue, Fascicle 1*, ed. D. Marin, Brasov, 2015, 45–50.

9 M. Obradović, B. Popkonstantinović and S. Mišić, "On the Properties of the Concave Antiprisms of Second Sort", *FME Transactions*. Vol. 41 No 3, (Belgrade), 2013, 256–263.

10 "Understanding 3D printed lattices: Performance and design", Fast Radius 2020. https://www.fastradius.com/resources/understanding-3d-printed-lattices-performance-and-design-considerations/ [accessed January 11. 2020.]

the structure lighter, at the same time retaining structural rigidity. Usually, such a network is made up of very simple polyhedral structures (cube, tetrahedron…), by their spatial packing (or 3D tessellation).

In this paper, for creating a 3D lattice, we use concave polyhedra of the second sort. Also, we extend the notion of a 3D lattice so to include shell lattice structures[11] composed of triangular plates which represent the faces of a polyhedron.

FORMING A 3D LATTICE PANEL

Although we can put together assorted members of the C-II-ns in various ways in order to form 3D lattices based on their close-packing[12], we focus on a single "layer" of such a structure, a panel-like 3D lattice. It is generated by multiplication of the chosen unit cell, the selected C-II-n representative, along the x-y directions. In the z direction the lateral surfaces form a deltahedral structure which makes the thickness of the panel (Fig. 1).

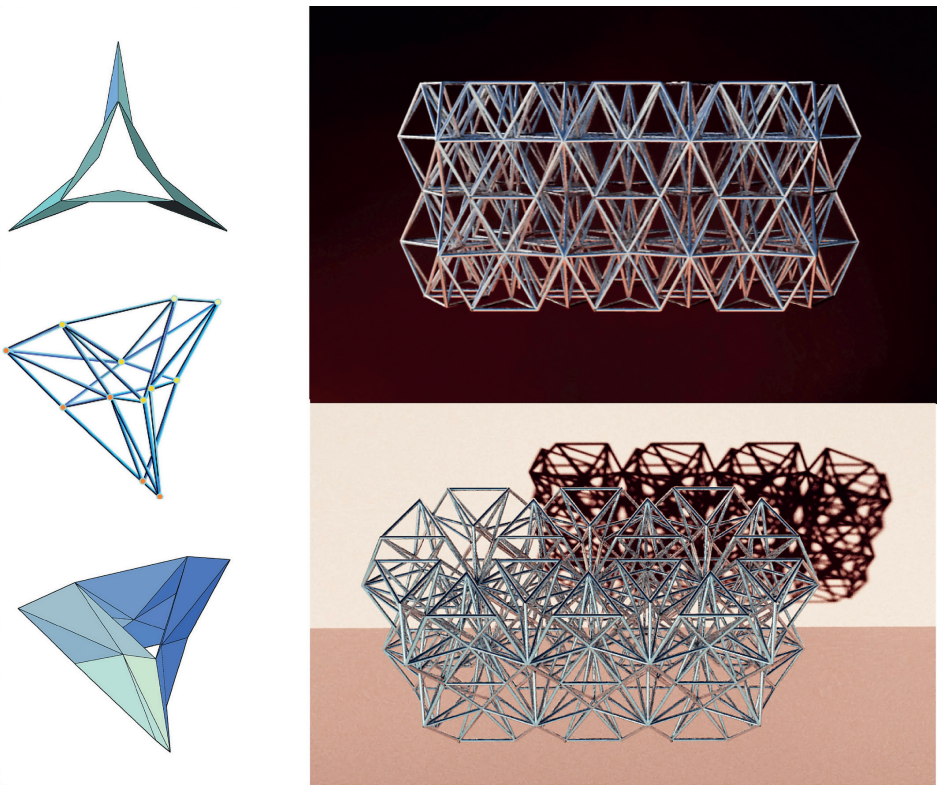

Fig. 2

11 T. Maconachie et al.,"SLM lattice structures: Properties, performance, applications and challenges", *Materials & Design*, 2019, 108–137.

12 P. Huybers, "Polyhedra for Building Structures, Why and How", in: *International Conference on Light-Weight Structures in Civil Engineering*, Warsaw, 1995, 25–29.

3D LATTICE PANEL AS A SPACE TRUSS MADE BY ARRANGEMENT OF C-II

In order to create a 3D lattice panel, for the framework of the structure we use a chosen representative of the concave polyhedra of the second sort (CA-II-3M, in the Fig. 2) as a unit cell. By self-multiplication of the unit cell in space we obtain a 3D lattice structure.

There are two ways to construct such a structure:

a) from rods that are continually linked one to another, building a spatial grid[13] [14],

b) from triangular plates (as in[15]) which represent the faces of the polyhedron.

In the case b, to obtain a lattice structure that lets the light pass through, we will omit the faces that represent the base polygons.

THE METHOD USED TO FORM A UNIT CELL

When we remove the base polygons from the C-II-nM representatives selected to be used in the 3D lattice formation, the unit cells become hollow, so they can create a honeycombed structure, more desirable for the purpose of application. Then, observed in 2D, by applying symmetry transformations, we form patterns similarly to the formation of wallpaper groups. In this way, we get visually interesting patterns in 2D, which transform into 3D lattice depending on the viewing angle.

The method we used is based on:

1. joining relevant **vertices** of the two adjacent units
2. connection of the **edges** of the two adjacent units
3. joining the **faces** of adjacent lateral surfaces so that they overlap.

USING 3D LATTICE PANELS AS A MATRIX FOR BRISE SOLEIL DESIGN

Brise soleil is an architectural element used on facades, over windows, as protection against sunlight and heat, while also playing the decorative role of an architectural ornament itself[16]. With the well-known effects that 2D patterns (of a brisesoleil) produce in creating cast shadows, an additional layer of light and shadow dynamics is now emerging. The light and shadows on the lateral faces, i.e. on the surfaces of these structures in 3D array, adds a new, unpredictable dimension to the aesthetics and ornamental character of these shapes. They look different depending on the time of day, the time of year, the lighting angle, etc. The impression changes even depending on the distance from the surface on which the shadows are cast.

GEOMETRIC ORNAMENT IN ARCHITECTURE

By development of new building technologies, especially parametric design, geometric ornament returns to the design of buildings on a large scale. Using the plays of symmetries with assigned modular tiles, a

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13 Z.Z. He, et al., "Mechanical properties of copper octet-truss nanolattices". *Journal of the Mechanics and Physics of Solids*, *101*, 2017, 133–149, Fig. 1.

14 T. Li, et al., "Exploiting negative Poisson's ratio to design 3D-printed composites with enhanced mechanical properties". *Materials & Design*, *142*, 2018. 247–258. Fig. 1.

15 T. Maconachie et al.,"SLM lattice structures: Properties, performance, applications and challenges", *Materials & Design*, 2019, 108137. Fig. 4.

16 S. Tolson, *Dictionary of Construction Terms*, 2014, 40.

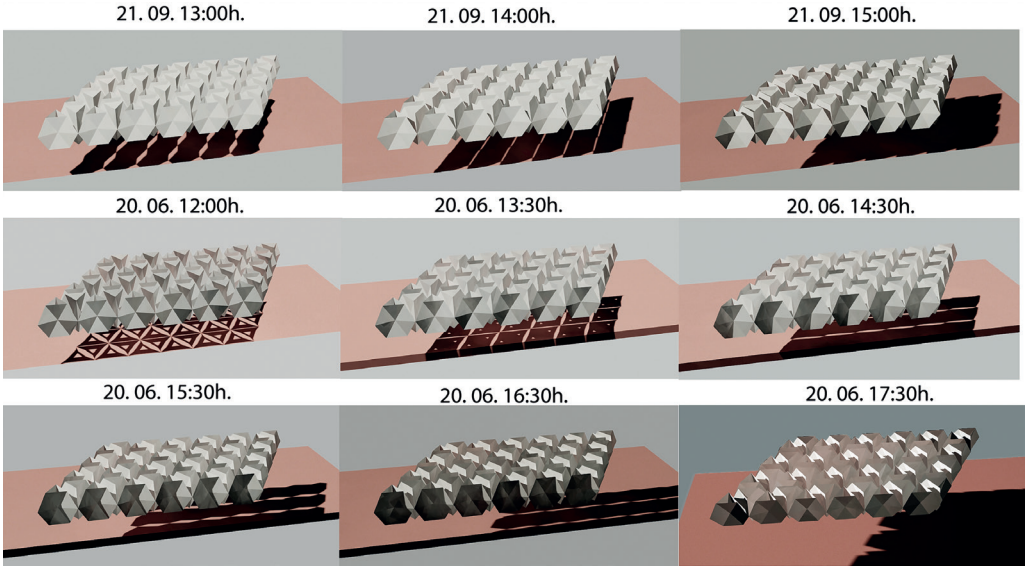

Fig. 3

simple geometric pattern can build a whole panel as an architectural element[17]. Such 3D patterns have already found a large application in architecture, so in this study we implement these experiences to new forms, introducing slightly more complex shapes, but equally simple to perform. We suggest their application as architectural elements: decorative panels, room dividers, brise soleil, etc., which change their appearance depending on the viewing angle. They can be performed in a quick and easy way, not only as 3D prints, but also as prefabricated or even folded elements.

THE APPEARANCE OF THE CAST SHADOW OF THE BRISE SOLEIL DEPENDING ON THE TIME AND DATE

Be it horizontal or vertical elements (e.g. pergolas or brise soleils), the influence of the angle of the sun rays on the appearance of the cast shadows is essential. In Fig. 3 we see what the shadows look like on the surface parallel to the 3D lattice panel, depending on the time of year and the time of day.

ORNAMENTAL 3D PATTERNS BASED ON CONCAVE POLYHEDRA OF THE SECOND SORT

In this section, we present key research findings: we provide a 3D lattice panels' layouts made up of selected C-II-nM representatives. We have adopted only one variation, major type, to illustrate the process, and to summarize numerous examples into a few essentials for the idea.

17 M. Stavric and S. Jablan, "Advanced geometry of modular tiles", in: *Generative Art conference GA 2011. Ed.* Roma, ed. C. Soddu, 2011, 344–357.

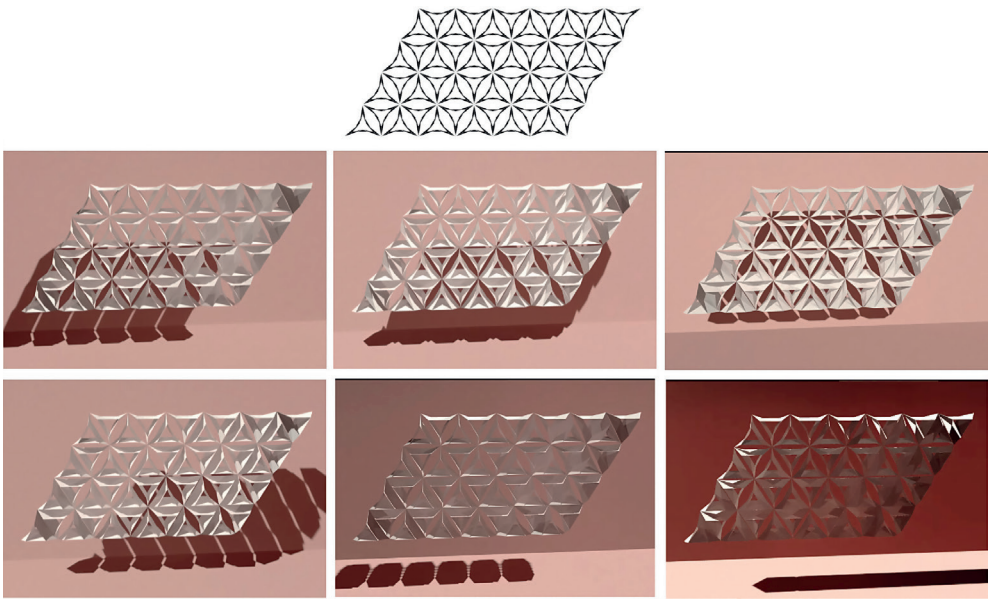

Fig. 4

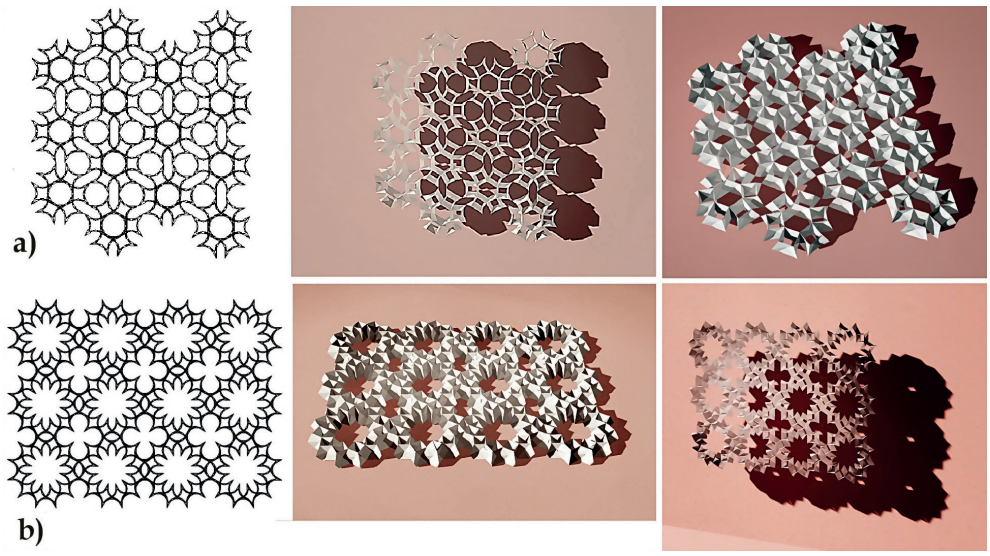

Fig. 5

1. Ornamental 3D patterns based on CA-II-3M

For a clearer understanding of the methods and adherence to the geometric principles of forming a 3D lattice panel in general, this research considers mainly the concave antiprisms of the second sort (CA-II-nM) with a few examples involving cupolae and pyramids.

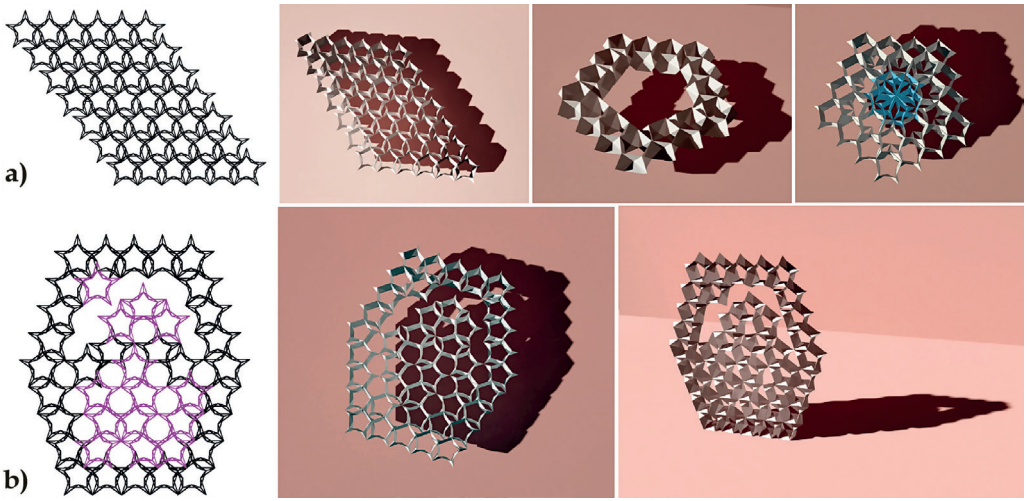

Fig. 6

In the first example, in Fig. 4, we see an arrangement of CA-II-3Ms touching by the vertices, thus building a pattern that corresponds to the hexagonal lattice[18].

2. Ornamental 3D patterns based on CA-II-4M

With an arrangement of CA-II-4Ms, we can obtain various patterns, some of which are shown in the Fig. 5. By the combination of two methods: touching by vertices and overlapping the faces, they are generating patterns that correspond to:

a) Hexagonal lattice (Fig. 5.a) having a pattern that resembles cantellated hexagonal tiling or rhombitrihexagonal tiling (3.4.6.4.)[19, 20]

b) Square lattice (Fig. 5b), having a pattern that resembles truncated square tiling (4.8.8.).

3. Ornamental 3D patterns based on CA-II-5M

With CA-II-5M we can obtain 3D patterns that correspond to:

a) Rhombic lattice (Fig. 6a)

a) Irregular lattice resembling the method of Penrose tiling[21] creation (Fig. 6b).

We can also play with shapes obtained so to form frames, rosettes, elements of irregular contours, etc. (Fig. 6).

4. Ornamental 3D patterns based on CA-II-6M, CC-II-6M and CP-II-6M

Using CA-II-6M we obtain 3D patterns that correspond to hexagonal lattice (Fig. 7). We can use various ways of 3D lattice panels creation just by using this single unit element. Here we see an example of

18 D. E. Joyce, "Lattices", web page of Clark University, Worcester, Massachusetts https://www2.clarku.edu/faculty/djoyce/wallpaper/lattices.html [accessed January 11. 2020.]

19 B. Grünbaum and G. C. Shephard, "Tilings by regular polygons", *Mathematics Magazine* Vol. 50. No5. (Washington D.C.), 1977, 227–247.

20 D.P. Chavey, "Tilings by Regular Polygons – II: A Catalog of Tilings", *Computers & Mathematics with Applications*. Vol. 17 No 1–2. Elsevier, 1989, 147–165

21 R. Penrose, "The role of aesthetics in pure and applied mathematical research", *Bull. Inst. Math. Appl.*, *10*, 1974, 266–271.

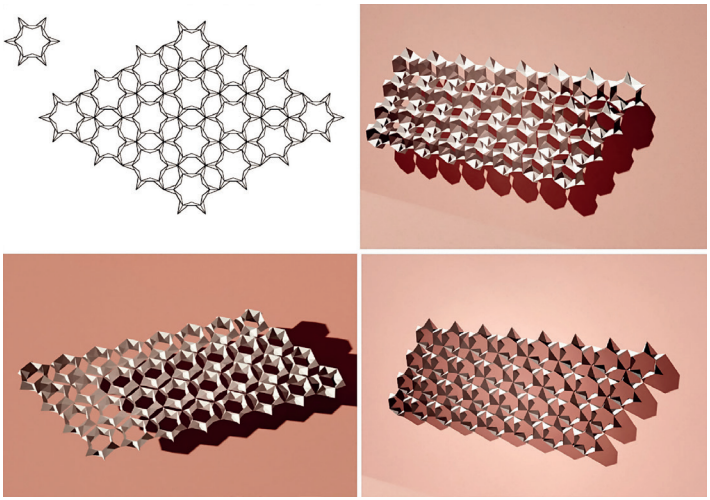

Fig. 7

connecting the cells by vertices, whereby a hexagonal 3D arrangement corresponds to ***p6m*** wallpaper group[22] and again resembles rhombitrihexagonal tiling, or *rhombihexadeltille*[23] (by Conway). Also, we can halve the thickness of such a panel, due to transverse symmetry of the CA-II. In doing so, we get a new appearance not only of the panel itself, but also of the cast shadows.

The example which uses CC-II-6M (given in Fig. 8) is the one with the edge connection of the 3D units. Thereby, the pattern obtained also corresponds to the hexagonal lattice and to the ***p6m*** wallpaper group,

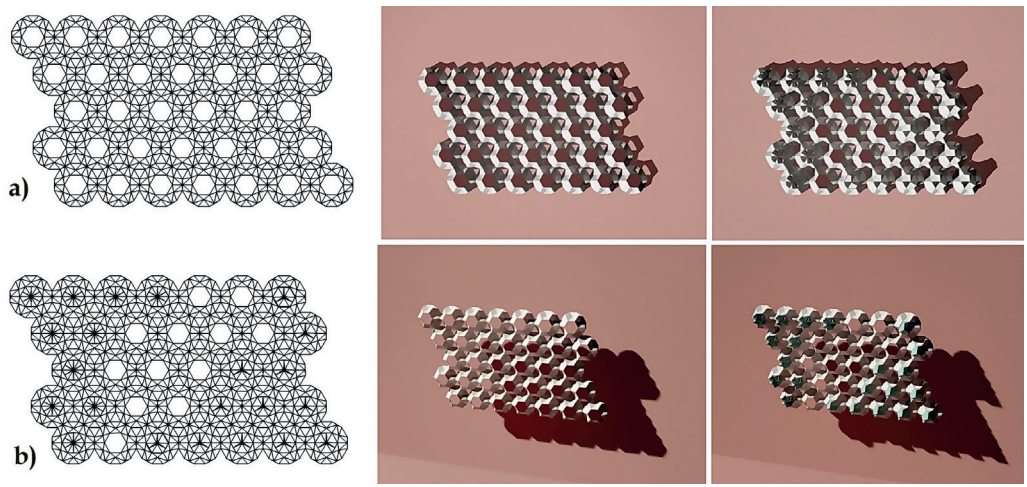

Fig. 8

22 D. E. Joyce, "Lattices", web page of Clark University, Worcester, Massachusetts https://www2.clarku.edu/faculty/djoyce/wallpaper/wall17.html [accessed January 11, 2020].

23 J. H. Conway et al., *The Symmetries of Things*, CRC books, Boca Raton, 2008, p.263.

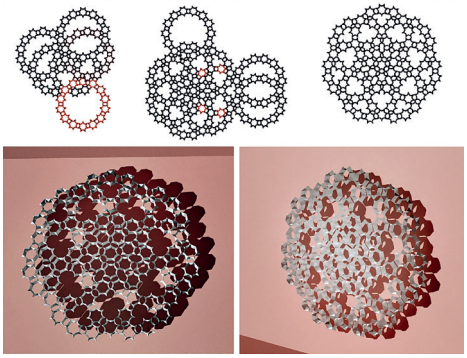

Fig. 9

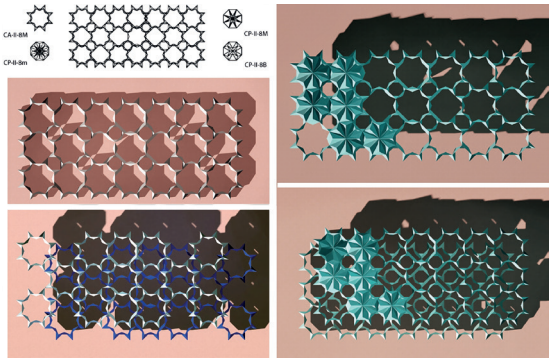

Fig. 10

but now resembles the truncated hexagonal tiling[24] or truncated hextille[25]. Moreover, we can supplement this structure with CP-II-6Ms[26], which add a new layer to the plastic, and have an additional role in blocking sunlight.

5. Ornamental 3D patterns based on CA-II-7M

The heptagon is more demanding for geometric combining compared to the previous bases. Tiling with this polygon is not possible, but different plays with arranging heptagons in patterns are. We use the CA-II-7M circuits (Fig. 9) which, by applying rotational symmetry, give a rosette-like 3D structure.

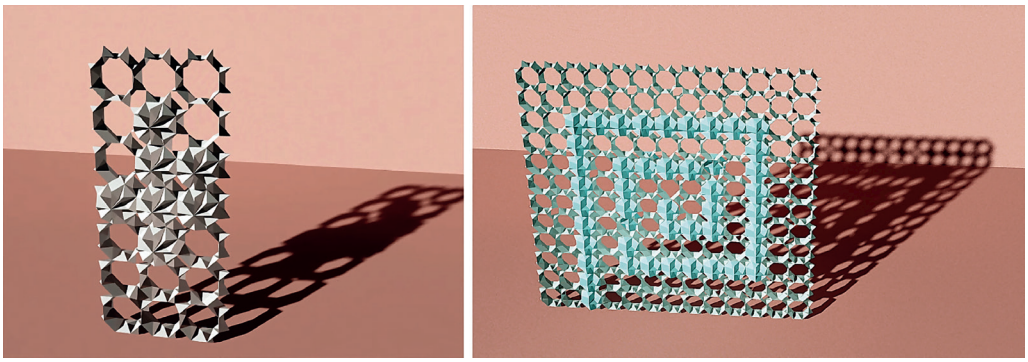

Fig. 11

24 B. Grünbaum and G. C. Shephard, "Tilings by regular polygons", *Mathematics Magazine* Vol. 50. No5. (Washington D.C.), 1977, 227–247.

25 J. H. Conway et al., *The Symmetries of Things*, CRC books, Boca Raton,2008.p. 263.

26 M. Obradović, S. Mišić and B. Popkonstantinović, "Concave Pyramids of Second Sort – The Occurrence, Types, Variations", in: *Proceedings of the 4th International Scientific Conference on Geometry and Graphics*, *moNGeometrija 2014*, ed. S. Krasić, Vol 2. M. Obradović, S. Mišić, B. Popkonstantinović, M. Petrović, B. Malešević, R. Obradović: Vlasina, 2014, 157–168.

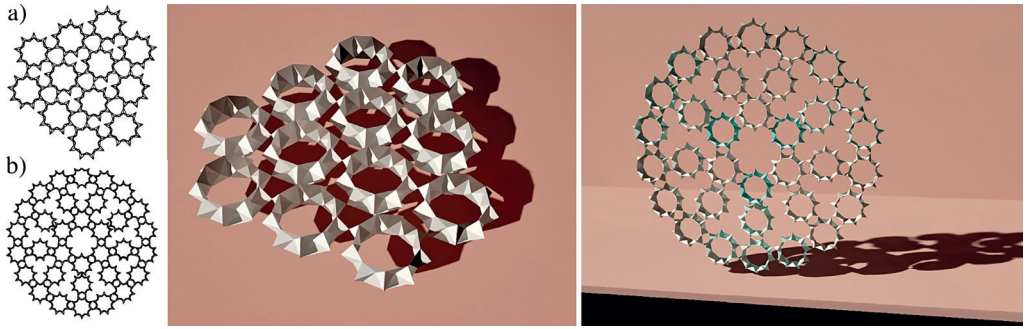

Fig. 12

Fig. 13

6. Ornamental 3D patterns based on CA-II-8M, CA-II-8M and CP-II-8B

Using CA-II-8M, we obtain 3D square lattice, resembling truncated square tiling. In Fig. 10 we see an example of connecting the cells by faces (the example used in[27]), whereby a hexagonal 3D arrangement corresponds to ***p4m***[28] wallpaper group. Also, we can double the panel by shifting one level of the panel eccentric to the lower level, thus getting another form of the structure itself. We can also add CP-II-8Ms to the matching octagonal openings, as in the example with CC-II-6M, in order to add new sun shades.

In a couple of examples, we propose models of such structures that can serve as a visually interesting ideas for design (or redesign) of the exterior and interior of the architectural objects (Fig. 11). Not only the defined structure itself, but also its fragments can be used for this purpose. In this way we can design fences, rum dividers, wall panels, etc.

7. Ornamental 3D patterns based on CA-II-9M, CC-II-9M and CP-II-9M

The nonagon has the same problem in plane tilling by regular, convex polygons, as heptagon does, but not by other type of concave polygons, or with non-regular polygons. Thus, in this example, the connection by vertices is the safest solution. We can also create a hexagonal lattice (Fig. 12a) and a rosette (Fig. 12b). Now, due to the looser fit of the nonagons, there is more space between the CA-II-9M unit cells, so the more light gets through this 3D lattice panel.

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27 M. Obradović, "Modularity of concave polyhedra of the second sort with octagonal bases", in: *Proceedings of the 18th International Conference on Geometry and Graphics. ICGG 2018. Advances in Intelligent Systems and Computing*, Vol. 809. No1. Ed. L. Cocchiarella, Milan, 2018, 942–954.

28 D. E. Joyce, "Lattices", web page of Clark University, Worcester, Massachusetts https://www2.clarku.edu/faculty/djoyce/wallpaper/wall11.html [accessed January 11, 2020].

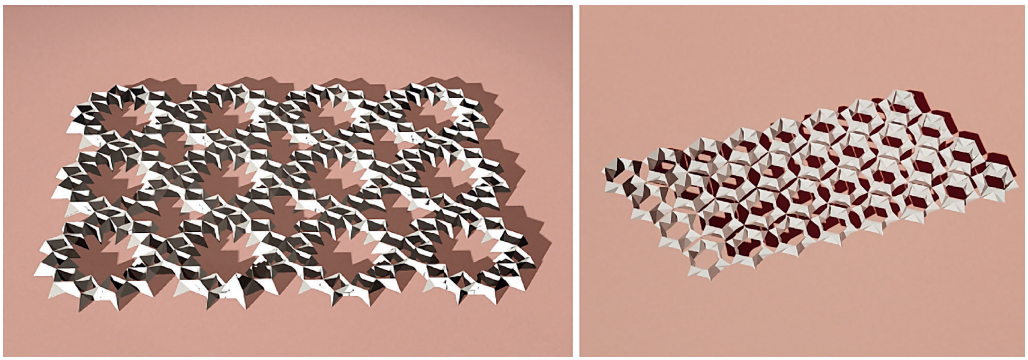

Fig. 14

In the example with CC-II-9Ms (Fig. 13) the edge-connection provides a pattern corresponding to the hexagonal lattice.We can also add CP-II-9Ms to the matching nonagonal openings, as in the example with CP-II-6, to add new sun shades and a new visual effect.

VARIATIONS

The thickness of the panel can be halved in some cases (the ones that use antiprisms), so we get a thinner structure with "face" and "back", having different tessellations of polygons emerging on them (Fig. 14). As an artistic intervention, these panels can be modified by joining deltahedral surfaces of other C-II-ns onto the compatible bases, whereby we add another layer of patterns to the resulting structure.

CLIMATE RESPONSIVE FACADES

3D patterns and lattices are currently experiencing real boom in the design and industry, thanks to the 3D printing capabilities. As for architecture, they can be applied not only as an element of ornamentation, but also as a functional component of the project, especially concerning climate responsive facades[29]. In Fig. 15, we see an example of Al Bahar Towers, Abu Dhabi, (by Aedeas Architects 2012.)[30] which shows the potential of deltahedral 3D lattice panels in its full splendor.

DIRECTIONS FOR FUTURE RESEARCH

Due to the simplicity of the C-II-ngeometry, 3D structures made as compositions of such unit cells are feasible and easy to perform in terms of production and assembly.

They are achievable not only with 3D printing, but can also be manually assembled or folded like origami, which allows the use of a much wider range of materials.

What makes an additional convenience in using the C-II-ns as unit elements is that their faces, regular polygons, allow and provoke various plays with symmetries. With a skillful composition of these patterns,

29 G. Shashank, "Climate responsive Building Façade", Behance.net https://www.behance.net/gallery/36375797/Climate-Responsive-Building-Facade Behance.net [accessed January 11, 2020]

30 K. Cilento "Al Bahar Towers Responsive Facade / Aedas", *Arch Daily*, September 05. 2012. archdaily.com https://www.archdaily.com/270592/al-bahar-towers-responsive-facade-aedas [accessed January 13, 2020]

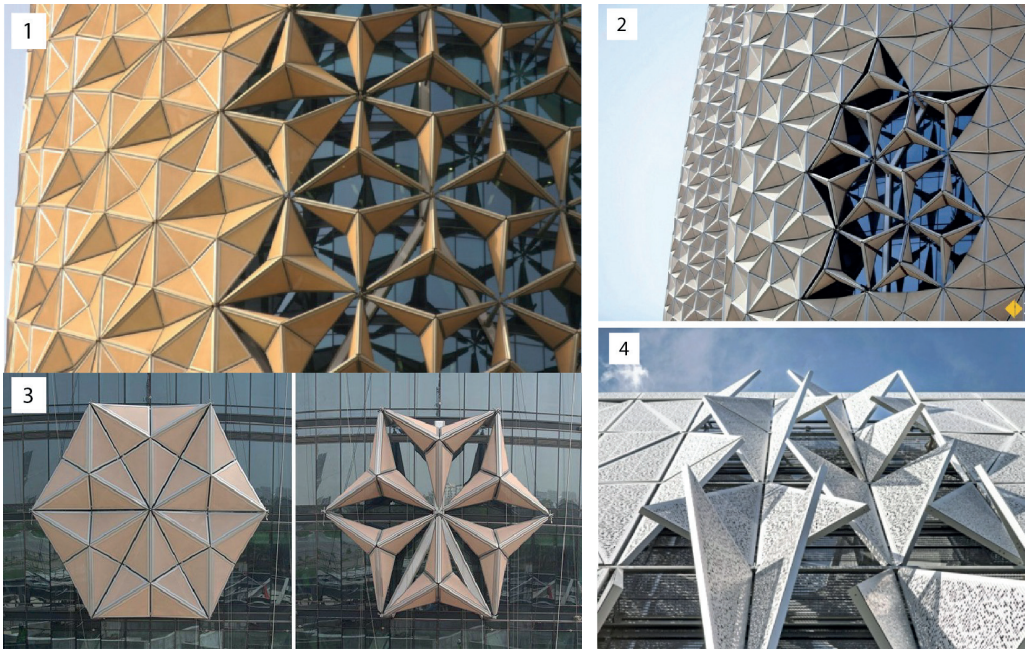

Fig. 15

and even by combining the members with congruent base polygons, but minding the symmetry of the pattern itself, an additional level of decorative design opens.

If we want to soften the sharp edges and introduce curved lines into these forms or do any other modification of the form, we can do this during modeling process, via tools of the software we use, and then produce the structure through 3D printing.

Examining the 3D lattices formed in the manner described above, along with their aesthetic dimension, the question of their static behavior arises. This is another issue that needs to be resolved.

One of the aims of the paper is to point out the possibilities offered by C-II-n geometry in artistic (visual and decorative) sense, linking the geometrical generation of 2D and 3D patterns with architectural design.

Acknowledgement

This paper is supported by the Ministry of Education, Science and Technology Development of Serbia, grant No. III44006.

ILLUSTRATIONS

1: 3D lattice and 3D lattice panel formed of CA-II-8Ms
3Д решетка и 3Д решеткасти панел састављен од CA-II-8M-ова
2: 3D lattice panel as a space truss made by arrangement of CA-II-3Ms
3Д решеткасти панел као просторна решетка сачињена распоредом више CA-II-3M-ова
3: The appearance of the cast shadow of the brise soleil based on the CA-II-3M depending on the time and date
Изглед бачених сенки брисолеја заснованих на CA-II-3M, у зависности од времена и датума

4: 3D lattice pattern based on the CA-II-3M and the shadows they cast depending on the time of day
Образац 3Д решетке заснован на CA-II-3M и сенке које баца у зависности од доба дана
5: 3D lattice pattern based on the CA-II-4M and the shadows they cast depending on the time of day, a) hexagonal lattice, b) square lattice
Образац 3Д решетке заснован на CA-II-4M и сенке које баца у зависности од доба дана, а) шестоугаона решетка, б) квадратна решетка
6: 3D lattice pattern based on the CA-II-5M and the shadows they cast, a) rhombic lattice, b) Irregular lattice resembling the Penrose tiling
Образац 3Д решетке заснован на CA-II-5M и сенке које баца у зависности од доба дана, а) ромбична решетка, б) неправилна решетка која подсећа на Пенроуз поплочавање
7: 3D lattice pattern based on the CA-II-6M and the shadows they cast, hexagonal lattice
Образац 3Д решетке заснован на CA-II-6M и сенке које баца, шестоугаона решетка
8: 3D lattice pattern based on the CC-II-6M and CP-II-6M, and the shadows they cast, a) truncated hexagonal tiling –like pattern, b) the same pattern filled with CP-II-6Ms
Образац 3Д решетке заснован на CA-II-6M и CP-II-6M и сенке које бацају; а) образац који наликује зарубљеном шестоугаоном поплочавању, б) исти образац попуњен CP-II-6Мовима
9: 3D lattice pattern based on the CA-II-7M and the shadows they cast
Образац 3Д решетке заснован на CA-II-7M и сенке које баца
10: 3D lattice pattern based on the CA-II-8M and the shadows they cast
Образац 3Д решетке заснован на CA-II-8M и сенке које баца
11: Ideas for design (or redesign) of the exterior and interior of the architectural objects
Идеје за дизајн (или редизајн) екстеријера и ентеријера архитектонских објеката
12: 3D lattice pattern based on the CA-II-9M, a) hexagonal lattice, b) rosette
Образац 3Д решетке заснован на CA-II-9M; а) шестоугаона решетка, б) розета
13: 3D lattice pattern based on the CC-II-9M and CP-II-9M
Образац 3Д решетке заснован на CA-II-9M и CP-II-9M
14: Variations of the 3D lattice panels – halved in thickness
Варијације 3Д решеткастих панела – преполовљене дебљине
15: The examples of climate responsive facades:

1. The Al Bahr Towers by Aedas Architects, Abu Dhabi, 2012. https://architizer.com/blog/practice/details/x-factor-construction/
2. The Al Bahr Towers by #AHR (former Aedas Architects), Abu Dhabi, 2012. https://twitter.com/tile_select/status/828655581271568384
3. Detail of Sun responsive façade, https://www.boredpanda.com/geometric-sun-shades-al-bahar-towers-abu-dhabi/?utm_source=google&utm_medium=organic&utm_campaign=organic
4. Henning Larsen's university building has a facade that moves in response to changing heat and light, https://www.dezeen.com/2015/07/14/henning-larsen-syddansk-universitet-sdu-kolding-campus-building-denmark-green-standards-university/

Примери фасада које реагују на климатске услове:

1. Куле Ал Бахр архитеката Аедаса, Абу Даби, 2012. https://architizer.com/blog/practice/details/x-factor-construction/
2. Куле Ал Бахр аутор #AXP (бивши архитекти Аедас), Абу Даби, 2012. https://twitter.com/tile_select/status/828655581271568384
3. Детаљ фасаде која реагује на сунце, https://www.boredpanda.com/geometric-sun-shades-al-bahar-towers-abu-dhabi/?utm_source=google&utm_medium=organic&utm_campaign=organic
4. Универзитетска зграда Хенинга Ларсена са фасадом која се креће као одговор на промену топлоте и светлости, https://www.dezeen.com/2015/07/14/henning-larsen-syddansk-universitet-sdu-kolding-campus-building-denmark-green-standards-university/

REFERENCES:

Chavey, Darrah P. "Tilings by Regular Polygons -II: A Catalog of Tilings", *Computers & Mathematics with Applications*. Vol. 17 No 1–2. Elsevier, 1989, 147–165.

Conway, John H., Burgiel, Heidi and Goodman-Strauss Chaim. *The Symmetries of Things, CRC* Books, Boca Raton, 2008.

Grünbaum, Branko and Shephard C. Geoffrey. "Tilings by regular polygons", *Mathematics Magazine* Vol. 50. No5 (Washington D.C.), 1977, 227–247.

He, Ze Zhou, Wang, Feng Chao, Zhu, Yin Bo, Heng, An Wu, & Park, S.Harold, "Mechanical properties of copper octet-truss nanolattices". *Journal of the Mechanics and Physics of Solids*, *101*, 2017, 133–149. https://ijat.ut.ac.ir/article_67595_498285fb58cc223e40ce660b73f74c55.pdf

Huybers, Peter. "Polyhedra for Building Structures, Why and How", in: *International Conference on Light-Weight Structures in Civil Engineering*, Warsaw, 1995, 25–29.

Joyce E. David. "Lattices", web page of Clark University, Worcester, Massachusetts https://www2.clarku.edu/faculty/djoyce/wallpaper/lattices.html [accessed January 11, 2020]

Li, T., Chen, Y., Hu, X., Li, Y., & Wang, L. "Exploiting negative Poisson's ratio to design 3D-printed composites with enhanced mechanical properties". *Materials & Design*, *142*, 2018, 247–258.

Maconachie, T., Leary, M., Lozanovski, B., Zhang, X., Qian, M., Faruque, O., & Brandt, M. "SLM lattice structures: Properties, performance, applications and challenges", *Materials & Design*, 2019, 108137.

Obradović, Marija, Mišić, Slobodan and Popkonstantinović, Branislav. "Concave Pyramids of Second Sort – The Occurrence, Types, Variations", in: *Proceedings of the 4th International Scientific Conference on Geometry and Graphics*, *moNGeometrija 2014,* ed. S. Krasić, Vol 2.: Vlasina, 2014, 157–168.

Obradović, Marija, Mišić, Slobodan and Popkonstantinović, Branislav. **"**Variations of Concave Pyramids of Second Sort with an Even Number of Base Sides", *Journal of Industrial Design and Engineering Graphics (JIDEG) – The SORGING Journal, Volume 10, Special Issue, Fascicle 1,* ed. D. Marin, University 'Transilvania' of Brasov / Romanian Society of Engineering Graphics SORGING, Brasov, 2015, 45–50.

Obradović, Marija**,** Popkonstantinović, Branislav and Mišić, Slobodan. "On the Properties of the Concave Antiprisms of Second Sort", *FME Transactions*. Vol. 41 No 3, (Belgrade), 2013, 256–263.

Obradović, Marija. "Modularity of concave polyhedra of the second sort with octagonalbases", *Proceedings of the 18th International Conference on Geometry and Graphics. ICGG 2018. Advances in Intelligent Systems and Computing*, Vol. 809. No1. Ed. L. Cocchiarella, Springer, Milan, 2018, 942–954.

Obradović, Marija and Slobodan Mišić. "Concave Regular Faced Cupolae of Second Sort", in: *Proceedings of 13th ICGG*, ed. G. Weiss, Dresden, 2008, El. Book: 1–10.

Penrose, Roger. "The role of aesthetics in pure and applied mathematical research", *Bull. Inst. Math. Appl*., 10, 1974, 266–271.

Stavric Milena and Jablan Slavik. "Advanced geometry of modular tiles", in: *Generative Art conference GA2011,* ed. C. Soddu, Roma, 2011, 344–357.

Tolson, Simon. *Dictionary of Construction Terms,* CRC Press, Boca Raton, 2014, 40.

"Understanding 3D printed lattices: Performance and design", Fast Radius 2020. https://www.fastradius.com/resources/understanding-3d-printed-lattices-performance-and-design-considerations/ [accessed January 11, 2020]

Shashank, Gautam. "Climate responsive Building Façade", Behance.net https://www.behance.net/gallery/36375797/Climate-Responsive-Building-Facade Behance.net [accessed January 11, 2020]

Cilento, Karen. Al Bahar Towers Responsive Facade / Aedas, Arch Daily, September 05. 2012. archdaily.com https://www.archdaily.com/270592/al-bahar-towers-responsive-facade-aedas [accessed January 13, 2020]

PHOTO SOURCES

https://architizer.com/blog/practice/details/x-factor-construction/ [accessed January 11, 2020]
https://twitter.com/tile_select/status/828655581271568384/photo/1 [accessed January 11, 2020]
https://www.boredpanda.com/geometric-sun-shades-al-bahar-towers-abu dhabi/?utm_source=google&utm_medium=organic&utm_campaign=organic [accessed January 11, 2020]
https://www.dezeen.com/2015/07/14/henning-larsen-syddansk-universitet-sdu-kolding-campus-building-denmark-green-standards-university/ [accessed January 11, 2020]

ABBREVIATIONS

C-II-n – concave polyhedron of the second sort
C-II-nM – concave polyhedron of the second sort, major type
C-II-nm – concave polyhedron of the second sort, minor
CA-II-n – concave antiprism of the second sort
CA-II-nM – concave antiprism of the second sort, major type
CA-II-nm – concave antiprism of the second sort, minor type
CC-II-nM – concave cupola of the second sort, major type
CC-II-nm – concave cupola of the second sort, minor type
CP-II-nM – concave pyramid of the second sort, major type
CP-II-nm – concave pyramid of the second sort, minor type
CP-II-nB – concave pyramid of the second sort, type B
p6m – wallpaper symmetry group 17 (lattice type:hexagonal)
p4m – wallpaper symmetry group 11 (lattice type:square)

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3Д РЕШЕТКАСТИ ПАНЕЛИ БАЗИРАНИ НА КОНКАВНИМ ПОЛИЕДРИМА ДРУГЕ ВРСТЕ: ИДЕЈЕ ЗА АРХИТЕКТОНСКЕ ОРНАМЕНТЕ

Резиме: Конкавни полиедри друге врсте могу се користити за стварање структуре која има карактеристике тродимензионалне решетке. Полиедри формирани у једном слоју генеришу решеткасте плоче које се могу користити као геометријски орнамент у архитектури са одређеним естетским квалитетима, попут високог нивоа симетрије. Испитали смо конкавне полиедре друге врсте (C-II-n) са базама од n=3 до n=9 да бисмо илустровали методу. Користили смо углавном CA-II-nM, јер су њихови омотачи погоднији за међусобно повезивање него полиедри са две различите базе, попут конкавних купола. Решеткаста структура панела добија се избацивањем правилних поледара њихових основа.
Сваки представник посматраног узорка конкавних полиедра друге врсте (C-II-n) може се третирати као јединична ћелија за креирање 3Д решеткасте плоче. Повезаност јединичних ћелија може бити врховима, ивицама или целом површином градивног једнакостраничног троугла. Неки од добијених 3Д образаца одговарају 2Д решеткама, квадратним, шестоугаоним и геометријама ромба, док они добијени од стране шестоугаоних и седмоугаоних представника одговарају розетама. Ове структуре могу се користити као архитектонски елементи који комбинују функцију и декорацију, као што су: брисолеји, вертикалне украсне плоче, преграде у ентеријеру и екстеријеру, ограде, перголе, итд. Због присутне модуларности, једнакостраничних троуглова као њихових градивних елемената, погодне су као могући дизајн за биоклиматске фасаде.
Кључне речи: делтаедар, решетка, антипризма, троугао, теселација, архитектура, брисолеји.